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TARGET PRIORITIZATION TO OPTIMIZE EXPECTED UTILITY FOR A SIMPLE BATTLE SCENARIO

ANN E. M. BRODEEN
DOUGLAS H. FRANK

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13. ABSTRACT *(Maximum 200 words)*

The U.S. Army Ballistic Research Laboratory has been researching innovative methodologies directly applicable to the problem of target value analysis (TVA) (i.e., assigning values to targets for the purpose of developing an optimal target engagement ordering). The problems associated with assigning target values as an aid in target selection have been examined by many approaches by other government agencies and contractors. Here the problem is examined from the standpoint of optimizing some utility function (i.e., maximizing some desired tactical objective). Expected values of utility functions are derived in terms of the two chosen research parameters, target threat and target vulnerability.

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1. INTRODUCTION

The problem of assigning values to targets as an aid in target selection has been examined for over a decade by many approaches, e.g., Fire Support Mission Area Analysis, U.S. Army Materiel Systems Analysis Activity (AMSAA) military worth study and classification tree methodology (another approach being pursued by the U.S. Army Ballistic Research Laboratory [BRL] [Brodeen and Winner 1988]). During the summer of 1988, Dr. Douglas H. Frank, Associate Professor, Department of Mathematics, Indiana University of Pennsylvania, worked with Ann E M. Brodeen, who is leading the BRL's target value analysis (TVA) investigation, to consider a probabilistic approach to the problem. (Dr. Frank was assigned to the BRL under the sponsorship of the U.S. Army Summer Faculty Research and Engineering Program.)

Target values are assessments keyed to the enemy's perception of the functions of its assets; TVA is the methodology that identifies potential high value target sets (i.e., assets the enemy threat commander requires for the successful completion of his mission) within the given tactical scenario. These targets, if successfully countered, can provide the friendly force with a tactical opportunity (U.S. Army Field Artillery School 1984). Although the TVA process may include complex algorithms, it should be simple enough for the user (i.e., the soldier) to understand. Simply put, he must be able to influence the process in order to meet the specific needs of his commander. For the field artillery to remain responsive, the soldier must be able to change target priorities as quickly as the tactical situation changes and be able to interpret the overall impact that such changes may have on the outcome of the operation.

Although TVA is a very subjective issue, the intent of this research was to show that assigned target values can be based on mathematical models. The following two objectives were defined for the proposed study: 1) define a value for each target in an enemy target array such that a sequence by which to engage the targets can be determined and 2) evaluate the target engagement sequence from the standpoint of optimizing an expected utility function based on a desired tactical outcome.

Details of the BRL's probabilistic approach to TVA based on a simple battle scenario are outlined in this report. Suggested areas for further research are also presented.

2. THE BATTLE

2.1 Parameter Selection. Subject matter discussions held with MAJ William T. Dougherty, Field Artillery Coordination Officer assigned to the BRL when the TVA probabilistic approach was initiated, led to the selection of target vulnerability and target threat as the parameters of interest. When considering an enemy target's value, it is natural to characterize this value by the ability of the friendly fire unit to remove the enemy target within some time frame (i.e., target vulnerability) as well as by the ability of the enemy target to achieve its objective within that same time frame (i.e., target threat). Removal of the enemy target is considered to be either its complete destruction or the infliction of a level of damage severe enough to abate the target's contribution to the enemy force given some particular tactical scenario. The objective of the enemy target might also be either the destruction of the friendly fire unit or the infliction of a severe level of damage upon it. (It should be noted that the definitions of the parameters developed by the principal investigators are in the interest of the research and may not be in accordance with those of the field artillery community.)

Since the parameters represent probabilities, each may take on values defined on the closed interval [0,1] (i.e., values ranging between 0 and 1, inclusive of the endpoints).

2.2 Mathematical Model. To obtain a probabilistic approach to TVA, consider a simple battle scenario, also referred to as a "gentlemanly" battle scenario, between a friendly fire unit and a group of T enemy targets, where $T \geq 2$. It is so-named due to the following specific assumptions and limitations imposed: 1) each enemy target as well as the friendly fire unit fires simultaneously and at the same rate of fire, 2) the probability of either removing the enemy target or of the friendly fire unit being removed does not change from volley to volley, and 3) all shots fired during each volley are independent. The strategy is to engage a single enemy target until it is removed before firing at the next target. The battle concludes when either the friendly fire unit has been removed or it has removed all T enemy targets (Frank 1988). For our purposes, a victory is defined as the removal of all T targets regardless of whether or not the friendly fire unit survives.

This process exemplifies an absorbing Markov Chain in which the absorbing states are characterized by the number of targets removed at the end of the battle. That is to say, if the process enters one of the states, the process can never leave that state (i.e., it is "absorbed" into that state). Rather than define an initial probability vector of the various starting states of the stochastic process and the transition matrix of probabilities of the process stepping from state to state, we chose to look at this process inductively (with a strong overtone of sequential statistics). Subsequently, various battle outcome probabilities can be derived for any integral number T of enemy targets, based on the threat and vulnerability of each target (Kemeny and Snell 1960). Consider the following parameters for each target $i = 1, \ldots, T$:

p_i = vulnerability of target i (probability of enemy target being removed)

$q_i = 1 - p_i$ (probability enemy target is not removed)

r_i = threat of target i (probability of friendly fire unit being removed by threat i)

$s_i = 1 - r_i$ (probability friendly fire unit is not removed by threat i)

$R_i = 1 - S_i$ (combined threat of remaining enemy targets after i - 1 removed)

$S_i = \prod_{\alpha=i}^{T} s_\alpha$ (probability friendly fire unit has not been removed after i - 1 enemy targets have been removed).

Suppose i - 1 enemy targets have been removed, where $i = 1, \ldots, T$. Define the following possible battlefield events:

A_i = removal of target i without the friendly fire unit being removed

B_i = removal of target i and the friendly fire unit

C_i = target i not removed but friendly fire unit removed.

Therefore,

$$P[A_i] = \frac{p_i S_i}{1 - q_i S_i}, \quad P[B_i] = \frac{p_i R_i}{1 - q_i S_i}, \quad P[C_i] = \frac{q_i R_i}{1 - q_i S_i}. \tag{1}$$

Equation 1 will hereafter be referred to as "Lemma 1." For the sake of brevity, only $P[A_1]$ will be proven since all other proofs are similar.

Let D_n be the event that enemy target 1 is hit (hit is synonymous with destroyed) on round n, and E_n be the event that the friendly fire unit is not hit on round n, where $n = 1, 2, \ldots$. Assuming that events D_n and E_n are independent, then $P[D_n] = p_1$ and $P[E_n] = S_1$. Since

$$A_1 = D_1E_1 \cup D_1^cE_1D_2E_2 \cup D_1^cE_1D_2^cE_2D_3E_3 \cup \ldots, \tag{2}$$

then

$$P[A_1] = p_1S_1 + q_1S_1(p_1S_1) + (q_1S_1)^2(p_1S_1) + \ldots = \frac{p_1S_1}{1 - q_1S_1}. \tag{3}$$

For h=1, ..., T, let U_h be the event that h targets are removed and the friendly fire unit has survived,

$$P[U_h] = \prod_{i=1}^{h} P[A_i] = \prod_{i=1}^{h} \frac{p_iS_i}{1 - q_iS_i}. \tag{4}$$

Equation 4 will subsequently be referred to as "Lemma 2;" its proof follows immediately from the Markov property of the battle described below.

Consider a finite stochastic process $\{X_t\}$. Think of $X_0, X_1, \ldots, X_{t-1}$ as "the past," X_t as "the present," and $X_{t+1}, X_{t+2}, \ldots$ as "the future" of the process relative to time t. The law of evolution of a stochastic process is often thought of in terms of the conditional distribution of the future given the present and past states of the process. In the case of a sequence of independent random variables or of a simple random "walk," for example, this conditional distribution does not depend on the past (i.e., the knowledge of the outcome of any preceding target engagement does not affect our predictions for the next target engagement).

For a *Markov process* we weaken this to allow the knowledge of the immediate past to influence our predictions. A finite Markov process is a finite stochastic process $\{X_0, X_1, \ldots, X_t, \ldots\}$ having the *Markov property* if, for each t and s, the conditional distribution of $X_{t+1}, \ldots, X_{t+s}$ given $X_0, X_1, \ldots, X_t$ is the same as its conditional distribution given X_t alone. For a Markov process, knowing the outcome of the last target engagement, we can neglect any other information we have about the past in predicting the future. It is important to realize that this is the case only if we know exactly the outcome of the last target engagement (Kemeny and Snell 1960; Bhattacharya and Waymine 1990).

Theorem 1 is now introduced. Recalling the definition of the end of the battle, and if H is the number of targets removed at the end of the battle,

$$P[H = h] = P[U_{h-1}] \cdot P[B_h] + P[U_h] \cdot P[C_{h+1}], \text{ for } h = 0, \ldots, T. \tag{5}$$

$$\text{Define } P[U_{-1}] = 0,\ P[U_0] = 1,\ P[B_0] = 0, \text{ and } P[C_{T+1}] = 1.$$

To prove Theorem 1, the battle ends with the friendly unit either removing or not removing the target, events D and D^c, respectively. Assuming $0 < h < T$, then

$$\begin{aligned} P[H = h] &= P[H = h \mid D] \cdot P[D] + P[H = h \mid D^c] \cdot P[D^c] \\ &= P[U_{h-1}] \cdot P[B_h] + P[U_h] \cdot P[C_{h+1}]. \end{aligned} \tag{6}$$

The special case of $h = 0$ can be obtained by observing that the event H = 0 is C_1. Also, the special case of H = T, our definition of victory, is presented as the following Corollary:

$$P[Victory] = \frac{\prod_{i=1}^{T} p_i s_i^i / s_T}{\prod_{i=1}^{T} (1 - q_i S_i)}. \tag{7}$$

The following proof is offered.

If victory is the event $U_{T-1} \cap (A_T \cup B_T)$, then

$$\begin{aligned} P[Victory] &= P[U_{T-1}] \cdot (P[A_T] + P[B_T]) \\ &= P[U_{T-1}] \cdot \frac{p_T}{1 - q_T S_T} \end{aligned}$$

$$= \frac{\prod_{i=1}^{T-1} p_i \prod_{i=1}^{T-1} S_i}{\prod_{i=1}^{T-1} (1 - q_i S_i)} \cdot \frac{p_T}{1 - q_T S_T} \qquad \text{by Lemma 2}$$

$$= \frac{\prod_{i=1}^{T} p_i \prod_{i=1}^{T} (s_i^i / s_T)}{\prod_{i=1}^{T} (1 - q_i S_i)}, \tag{8}$$

where it is observed that

$$\prod_{i=1}^{T-1} S_i = (s_1 \cdot s_2 \cdot \ \ldots \ \cdot s_T)(s_2 \cdot s_3 \cdot \ \ldots \ \cdot s_T) \ \ldots \ (s_{T-1} \cdot s_T) = s_1 s_2^2 s_3^3 \ \ldots \ s_{T-1}^{T-1} s_T^{T-1} . \tag{9}$$

3. TARGET VALUE APPROACHES

The philosophy behind the probabilistic approach is one of evaluating the impact of reducing the overall threat from the enemy target array, which may be represented by any mix of target types, on the ultimate goal of total victory. More specifically, we pose the following questions, "Which targets should be attacked to ..."

- maximize the probability of victory?
- increase the probability of victory?
- reduce the overall threat?

See Figure 1.

"Which targets should be attacked to ..."

- maximize the probability of victory?
- increase the probability of victory?
- reduce the overall threat?

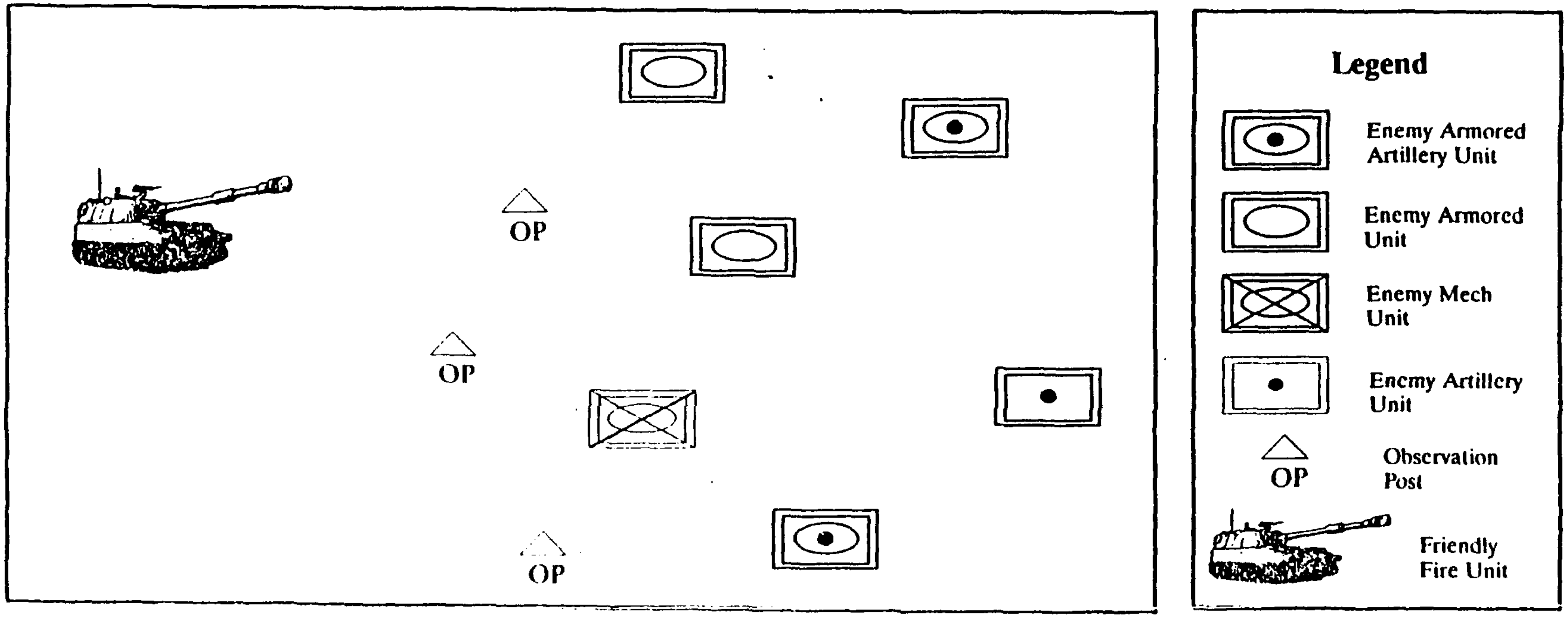


Figure 1. Probabilistic Target Value Concept.

It was shown in Section 2.2 that battlefield events could be expressed in terms of a mathematical model. Four different target value algorithms are derived based on those event probabilities. These algorithms will be coupled with the utilities developed in Section 4 to assess various target engagement schemes.

3.1 <u>Target Value One</u>. Assume a single friendly fire unit engages two enemy targets: T = 2. Let V_1 be the event that victory occurs if the target engagement ordering is 1, 2, while V_2 is the event that victory occurs if the ordering is 2, 1. Recalling Equation 7, then

$$\frac{P[V_1]}{P[V_2]} = \left(\frac{p_1 p_2 s_1 s_2}{(1 - q_1 s_1 s_2)(1 - q_2 s_2)} \right) \Big/ \left(\frac{p_2 p_1 s_2 s_1}{(1 - q_2 s_2 s_1)(1 - q_1 s_1)} \right)$$

$$= \frac{(1 - q_2 s_2 s_1)(1 - q_1 s_1)}{(1 - q_1 s_1 s_2)(1 - q_2 s_2)} . \tag{10}$$

To obtain a prioritization value for target 1, regardless of the other target, let $s_2 \rightarrow 0$ in the above ratio, then

$$1 - q_1 s_1 = p_1 + r_1 - (p_1 \cdot r_1). \tag{11}$$

Thus, target 1 is compared to the ultimate threat which, in this case, is represented by target 2 (*i.e., since* $s_2 \rightarrow 0$, *then* $r_2 \rightarrow 1$). Intuitively, if T = 2 and it is assumed that target 2 poses a much greater threat than target 1, the <u>probability of victory is maximized</u> given the target engagement ordering 1,2. Why? Presumably, the friendly fire unit will survive the encounter with target 1 but not necessarily the encounter with target 2. Given that our definition of victory calls for the removal of all enemy targets, regardless of whether or not the friendly fire unit survives, the opportunity for total victory in the two target argument is greater if the friendly fire unit takes out the "easier" target initially, surviving to engage the more difficult (to remove) target.

Given a target with vulnerability P and threat R, the first derived target value is defined as

$$VAL\ 1 = P + R - (P \cdot R). \tag{12}$$

This function is increasing in both P and R and is scaled from 0 to 1. Its obvious appeal lies in its simplicity.

3.2 Target Value Two. Simple battle simulations and later probability calculations implied that a target's value is not necessarily symmetric in both vulnerability and threat, but rather threat should carry more weight. This led directly to the second approach of considering the increase in the probability of victory after a target is removed:

$$\frac{P[\textit{Victory without the target in the battle}]}{P[\textit{Victory with all targets in the battle}]}.$$

Recalling Equation 7 and the Markov property, this ratio is

$$1/P[A_1] = \frac{1 - q_1 s_1 S_2}{p_1 s_1 S_2} = \frac{S_2^{-1} - q_1 s_1}{p_1 s_1}. \tag{13}$$

The value of a target could be this increase in the probability of victory multiplied by p_1.

For direct comparison with VAL 1, suppose $S_2^{-1} = 1$, (i.e., therefore, the combined threat of the remaining targets once the first target is removed is $1 - S_2 = R_2 \to 0$) then

$$VAL\ 2 = \frac{P + R - (P \cdot R)}{1 - R}. \tag{14}$$

This approach delegates more weight to threat while maintaining as simple a format as VAL 1. However, it has an unbounded scale.

3.3 Target Value Three. Additional simulations seemed to further imply that an individual target's value cannot be divorced from the overall threat posed by the combined target array. Consider the reduction in the overall threat if a particular target is removed. Recall that $S_1 = s_1 \cdot s_2 \cdot \ldots \cdot s_T$. Then for each target *i*, let $F_i = 1 - S_1/s_i$, where F_i represents the combined threat of all the targets except *i*. The relative decrease in the overall threat with target *i* removed becomes

$$\frac{R_1 - F_i}{R_1} = 1 - \frac{F_i}{R_1}. \tag{15}$$

Multiplying this decrease by p_i yields the third target value algorithm:

$$VAL\ 3 = p_i\left(1 - \frac{F_i}{R_i}\right). \tag{16}$$

VAL 3 is increasing in both vulnerability and threat and is scaled between 0 and 1. It, too, gives more weight to threat than does VAL 1.

3.4 <u>Target Value Four</u>. Suppose T enemy targets are acquired in a particular order, where T > 2. The final target value approach is a natural extension to VAL 1 as it attempts to select a <u>target sequence that maximizes the probability of victory</u>. Let $P_1(V)$ be the probability of victory in that order and let $P_2(V)$ be the probability of victory if target 1 and target 2 of the original sequence are transposed.

We now state Lemma 3:

$$P_1(V) > P_2(V),$$

if and only if

$$s_2 + S_1 q_2 r_2 > s_1 + S_1 q_1 r_1. \tag{17}$$

The proof of Lemma 3 is as follows. Let

$$p_1^* = p_2,\ p_2^* = p_1, \text{ and } p_i^* = p_i, \quad \text{where } i > 2,$$

and

$$r_1^* = r_2,\ r_2^* = r_1, \text{ and } r_i^* = r_i, \quad \text{where } i > 2. \tag{18}$$

Define q_i^*, s_i^*, and S_i^* in the usual manner. Note that $S_i^* = S_i$ for all $i \neq 2$ and $S_2^* = \frac{S_1}{s_2}$.

From Equation 7,

$$P_1[V] = \frac{\prod_{i=1}^{T} p_i\left(s_i^I/s_T\right)}{\prod_{i-1}^{T}(1-q_iS_i)}, \quad \text{and} \quad P_2[V] = \frac{\prod_{i=1}^{T} p_i^*\left(s_i^{I*}/s_T^*\right)}{\prod_{i-1}^{T}(1-q_i^*S_i^*)}. \tag{19}$$

Thus, $\frac{P_1[V]}{P_2[V]}$ can be reduced to

$$\frac{P_1[V]}{P_2[V]} = \frac{s_2}{s_1}\,\frac{(1-q_2S_1)\left(1-q_1\dfrac{S_1}{s_2}\right)}{(1-q_1S_1)(1-q_2S_2)} = \frac{(1-q_2S_1)(s_2-q_1S_1)}{(1-q_1S_1)(s_1-q_2S_1)}. \tag{20}$$

$\frac{P_1[V]}{P_2[V]} > 1$ is equivalent to

$$s_2 - q_2S_1s_2 - q_1S_1 + q_1q_2S_1^2 > s_1 - s_1q_1S_1 - q_2S_1 + q_1q_2S_1^2,$$

or

$$s_2 + q_2S_1(1-s_2) > s_1 + q_1S_1(1-s_1),$$

or

$$s_2 + S_1q_2r_2 > s_1 + S_1q_1r_1. \tag{21}$$

The algebra holds true for any pair of adjacent targets (n - 1, n), for n = 2, ..., T.

We now state Theorem 2: the target ordering which yields a maximum probability of victory is such that, for any pair of adjacent targets (n-1, n),

$$s_n + S_{n-1} q_n r_n > s_{n-1} + S_{n-1} q_{n-1} r_{n-1}, \quad \text{for } n = 2, \ldots, T. \tag{22}$$

If for any pair of adjacent targets the inequality fails, the probability of victory is increased by interchanging the target orderings for that particular pair. Given an enemy target with vulnerability P and threat R, Theorem 2 allows us to define the final target value approach as

$$VAL\ 4 = [(1 - R) + K \cdot (1 - P) \cdot R]^{-1}. \tag{23}$$

VAL 4 is increasing in both P and R; it is scaled between 1 and ∞.

The value $K < 1$ (a heuristic constant) is dependent on the particular target array being engaged. Although K may be chosen in several ways, we use,

$$K = \prod_{i=1}^{T} s_i = S_1 . \tag{24}$$

From the above, it should be obvious that the greater the number of targets in the array, the smaller the value of K. In fact, if the threat of the initial target to be engaged is very large, only the (1-R) term of the value algorithm should be stressed. Ideally, we would like $K \leq .5$.

Suppose we can assume that all the targets in the array are identical. To get a value which depends only on T, the number of targets in the array, we could use

$$K = (1 - R)^T, \tag{25}$$

where the product of the s_i's associated with each target could be replaced by some estimate (e.g., the geometric mean).

4. EVALUATION CRITERIA

In general, decision makers such as gamblers, baseball managers, insurance companies, and others engage in what is colloquially referred to as "playing the percentages," characterized by a preference for the optimal act that yields the greatest long-run average profit. That is, the optimal act is the one that would result in the largest long-run average profit if the same decision were to be made repeatedly under the same conditions; as the number of repetitions becomes large, the observed average payoff approaches the theoretical expected payoff. However, many important decisions are made under unique sets of conditions, and in some occasions it may not be realistic to think in terms of many repetitions of the same decision situation. Indeed, many of the field artillery commander's most important decisions are unique, high-risk situations, whereas less important, routine decisions are ones that may be delegated to subordinates. Therefore, it is useful to have an apparatus for dealing with one-time decision making.

Utility theory provides such an apparatus, as well as providing a logical method for repetitive decision making. The term "utility" as conceived by von Neumann and Morgenstern (1947) is a measure of value used in the assessment of situations involving risk, which provides a basis for decision making. Different sets of axioms that imply the existence of utilities with the property that expected utility is an appropriate guide for consistent decision making are presented in von Neumann and Morgenstern (1947), Savage (1954), Luce and Raiffa (1957), Pratt, Raiffa, and Schlaifer (1965), and Fishburn (1970).

4.1 Construction of Utility Functions. The different algorithms for determining target values do not always yield the same target engagement ordering. This poses the obvious question: which approach should be used? The desired approach would be the one whose target ordering provides the "best" result. In terms of "victory" this seems to be VAL 4. However, if T, the total number of enemy targets, is large, then "victory" for a single friendly fire unit would quite likely be a rare event. Thus, additional criteria shall be considered for assessing "best" results.

Recall that the overall objective is to assign a value to each enemy target to determine the order in which to engage the targets. This order should be chosen to maximize some desired battle result. Therefore, consider a utility function, U, of the number of targets removed, H, by the friendly fire unit during the battle. This function should depend on the battlefield scenario as well as the desired battle objective of the friendly fire unit. Assume that U(H) will be non-decreasing, U(0) = 0 and U(T) = 1.

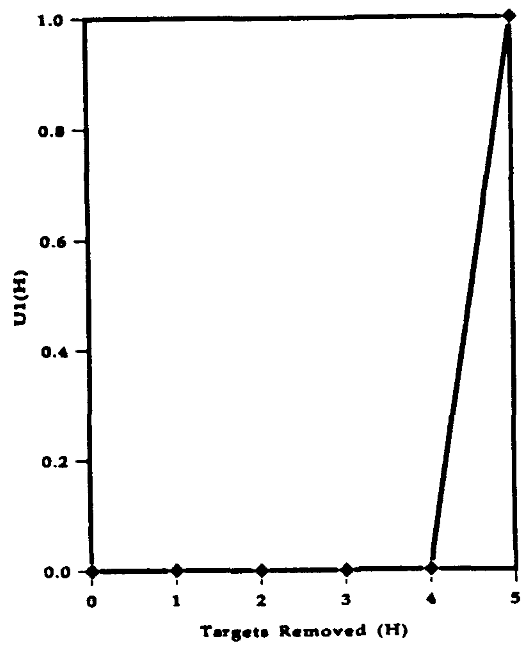
Generally, U(H) is assigned over a continuous range of possibilities; however, special liberty has been taken in the analysis of the utility functions discussed below. Since each of these utility functions is based on the mathematical model's assumption that an enemy target either survives or is completely removed from the battle, these functions are evaluated only at discrete points. The expected utility, evaluated as a discrete function, is

$$\sum_{h=1}^{T} U(H)\, p_H(h) \, ,$$

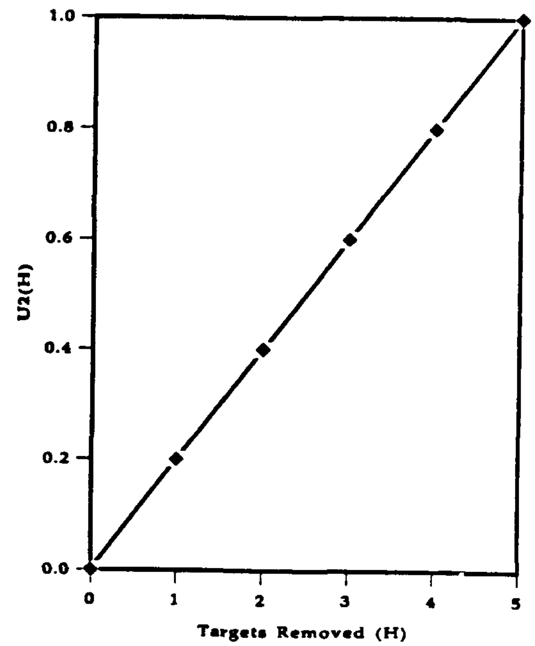
where $p_H(h) = P[H = h]$. Future consideration of enemy target fractional damage rather than complete removal of the target would allow these same utilities to be evaluated as continuous functions.

Four types of utility functions are considered: 1) concave upward; 2) linear; 3) concave downward; and 4) S-shape. The concave downward function rewards a few hits, whereas the concave upward function is weighted toward "victory." An S-shape, or inflective, function is useful if the goal of the friendly fire unit is to destroy a given fraction of the enemy targets. It is expected that the preferred target values will be based on the relevant utility function. Figures 2a–2d depict each of the functions for H = 0, 1, ..., 5 targets. Note that the functions are drawn as continuous curves only for the purpose of illustration.

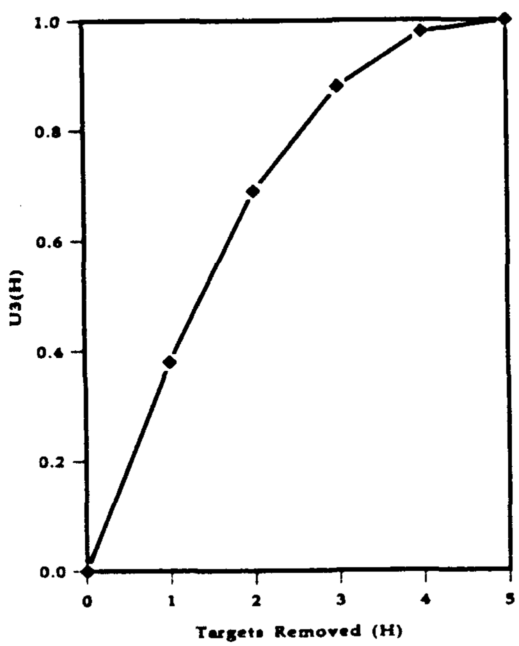
4.2 Utility Based on Total Victory. The first utility function, U1(H), is based on total victory (i.e., removal of all enemy targets) and is an extreme example of a concave upward function (see Figure 2a).



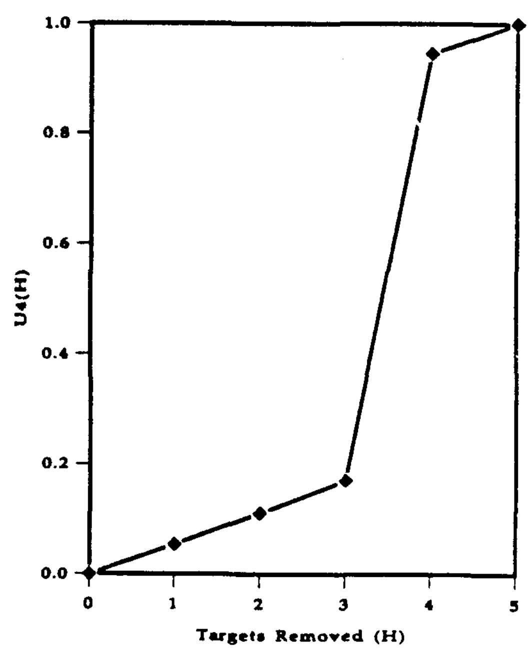
a. Utility for Total Victory.



b. Utility for Number of Targets Removed.



c. Utility for Reduction in Threat.



d. Utility for Force Reduction.

Figure 2. Types of Utility Functions.

$$U1(H) = \begin{cases} 0, & \text{if } H < T \\ 1, & \text{if } H = T \end{cases}. \tag{26}$$

4.3 Utility Based on the Number of Targets Removed. The number of enemy targets removed can be represented by a linear utility function, U2(H) (see Figure 2b).

$$U2(H) = H/T. \tag{27}$$

4.4 Utility Based on a Reduction in Threat. If H targets are removed, the overall enemy threat is reduced. If the targets are prioritized according to VAL 3, the third utility, U3(H), is a concave function (see Figure 2c).

$$U3(H) = 1 - \frac{R_{H+1}}{R_1}, \tag{28}$$

where R_i is defined as before. Note that U3(H) is based on a specific target engagement ordering, whereas utilities 1, 2, and 4 are not.

4.5 Utility Based on a Reduction in Force. During a particular battlefield scenario, the friendly fire unit might be given responsibility for the removal of a certain fraction (call it F) of the target array. Removal of a smaller fraction than F leads to a small utility while removal of a larger fraction yields a higher utility (see Figure 2d).

$$U4(H) = \begin{cases} C^{H/T}-1, & \text{if } H \leq F \cdot T \\ 2-C^{1-H/T}, & \text{if } H > F \cdot T \end{cases}, \tag{29}$$

where C is chosen such that $C^F - 1 \leq .5$. Consider the following example:

$$U4(H) = \begin{cases} (1.3)^{H/T}-1, & \text{if } H \leq .7T \\ 2-(1.3)^{1-H/T}, & \text{if } H > .7T \end{cases},$$

where we let T = 5 and F = .7. Computed values of U4(H) for the example are given in Table 1.

Table 1. Values for U4(H)

H	0	1	2	3	4	5
U4(H)	.0000	.0539	.1107	.1705	.9461	1.0000

5. COMPARISON OF VALUES AND UTILITIES BY EXAMPLE

The preferred target value is based on the relevant utility function. Consider the following:

- Theorem 2 dictates that we should expect VAL 4 to be best for U1. Examples appear to agree with this.

- Given that U2 weights each hit the same, an ordering scheme which yields a high probability of some hits is recommended. Since P[H > 0] can be shown to be a maximum in the order of the P_i, VAL 1 or VAL 2 should perform best. The advantage of VAL 1 is its simplicity. (Note: when H = 0, no targets have been hit. We wish to rank the targets in order of the most vulnerable to the least vulnerable, i.e., when we increase the P_i, we increase the probability of a hit. The P[H > 0] may be generated by the negative binomial distribution.)

- By definition, one would expect VAL 3 to be best for U3.

- Since U4 tends toward large H values, VAL 4 might be expected to be best. However, examples have been inconclusive.

5.1 Example With Six Targets. Table 2 illustrates the statement that different target value algorithms do not always generate the same target engagement ordering (see Section 4.1). It also illustrates the observations outlined previously.

Note that the values derived under each approach have been scaled by multiplying by 100. The vulnerability and threat estimates are arbitrary.

Table 2. Computed Target Values

Target	Vulnerability	Threat	100x VAL 1	100x VAL 2	100x VAL 3	100x VAL 4
A	.30	.10	37.00	41.11	2.04	107.92
B	.25	.01	25.75	26.01	.15	100.72
C	.15	.18	30.30	36.95	2.01	113.88
D	.10	.28	35.20	48.89	2.38	122.59
E	.05	.05	9.75	10.26	.16	103.30
F	.01	.24	24.76	32.58	.19	117.61

Table 3 presents the orders of engagement based on the target values computed for Table 2. Since the distribution of H (number of targets removed during the battle) is known, the expected value of each utility function for each order of engagement can be calculated. These expected values are also given in Table 3. (The BAD order of engagement is simply the reversal of VAL 2, a "good" ordering. RANDOM represents an arrangement based on random throws of a die. The VULNERABILITY and THREAT orderings come from ranking the estimates of Table 2.) Values of C = 1.3 and F = .7 were used for the calculation of E[U4].

Table 3. Target Orderings and Associated Utilities

Order		100x E[U1]	100x E[U2]	100x E[U3]	100x E[U4]
VULNERABILITY	ABCDEF	.109	7.859	2.920	2.124
THREAT	DFCAEB	4.321	2.560	3.659	.690
VAL 1	ADCBFE	.925	7.298	3.510	1.966
VAL 2	DACFBE	2.538	3.065	3.971	.833
VAL 3	DACFEB	2.658	3.064	3.971	.831
VAL 4	DFCAEB	4.321	2.560	3.659	.690
RANDOM	DBCFAE	1.098	2.989	3.706	.810
BAD	EBFCAD	.042	1.482	1.261	.400

It is interesting to note that the best order for U1 is based on either THREAT or VAL 4. For U2, the best arrangement is based on VULNERABILITY; however, the target engagement ordering based on VAL 1 does almost as well.

The target engagement orderings of both VAL 2 and VAL 3 perform best for U3. (It can be seen that orders of engagement based on VAL 2 and VAL 3 are very similar in this example.) No trend is apparent for U4.

The BAD arrangement performs poorly regardless of the utility. Also, arrangements based on some target value approach perform better than the RANDOM ordering in almost all aspects.

5.2 Example With Three Targets. Table 4 presents target values derived utilizing the four approaches for each of three enemy targets exhibiting the following characteristics: target A has high vulnerability, low threat; target B is balanced; target C has low vulnerability, high threat. The estimates associated with vulnerability and threat are arbitrary. As in the previous example, the target values have been scaled by multiplying by 100. Table 5 considers all possible arrangements of the three targets and the expected utilities associated with each arrangement. Values of .00, .15, .75, and 1.00 were used for E[U4(H)], where H = 0, 1, 2, and 3 targets removed, respectively.

Table 4. Computed Target Values

Target	Vulnerability	Threat	100x VAL 1	100x VAL 2	100x VAL 3	100x VAL 4
A	.25	.05	28.75	30.26	2.02	102.81
B	.15	.15	27.75	32.65	4.06	107.85
C	.05	.25	28.75	38.33	2.56	111.88

Table 5. Target Orderings and Associated Utilities

Order	100x E[U1]	100x E[U2]	100x E[U3]	100x E[U4]
ABC	1.007	1.863	6.932	1.257
ACB	1.212	1.684	6.058	.928
BAC	1.246	1.407	10.123	1.097
BCA	1.795	1.187	10.350	.682
CAB	1.973	.609	7.264	.497
CBA	2.395	.586	7.563	.441

E[U1] increases as targets of higher VAL 4 are interchanged. E[U2] varies in the order of VULNERABILITY. Here, the maximum values of E[U3] are for engagement orderings based on VAL 3. Once again, no significant trend is apparent for E[U4].

6. CONCLUSIONS

Each of the target value algorithms derived and analyzed has interesting features. If the desired battle objective is to remove as many targets as possible, then VAL 1 appears to be best. VAL 2 appears to be the least useful of the algorithms developed. Its denominator distorts the values in extreme cases, $R \to 1$, with the implication that a target may be reported as more valuable than it should be. If the goal is to inflict as much damage as possible on the enemy, as measured by U3, then VAL 3 seems most appropriate. Unfortunately, VAL 4, which almost always gives optimal results when considering a complete victory, does not perform well for other considerations.

One of the obvious needs is a method for acquiring accurate values for the vulnerability and threat parameters. These values not only depend on inherent target characteristics but also on the battlefield conditions and the objectives assigned to the friendly fire unit. Initially, the literature could be perused for probabilities of hit and kill. One promising statistical approach would be to utilize the CART software (Classification and Regression Trees), with input in the form of experimental data, simulated data, and officers' judgements (Brodeen and Winner 1988; Dougherty and Kaste 1988).

Theorem 2 is the only derived result relating to optimality, and it is very weak. Additional conditions for optimality of U1, U2, and U3, as well as other utility functions, should be developed.

The battle scenario is rather simplistic. Indeed, the battle may be criticized since it assumes the friendly fire unit has only one weapon, the removal of which terminates the battle. More sophisticated simulations should be developed, and the results from all models should be compared.

The target value algorithms and evaluation criteria presented in this paper may be used but should be regarded only as a first step in the development of optimal target engagement orderings.

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